edge effect is possible. Let us note that the edge effect in the case of isotropic shells is always nondegenerate.

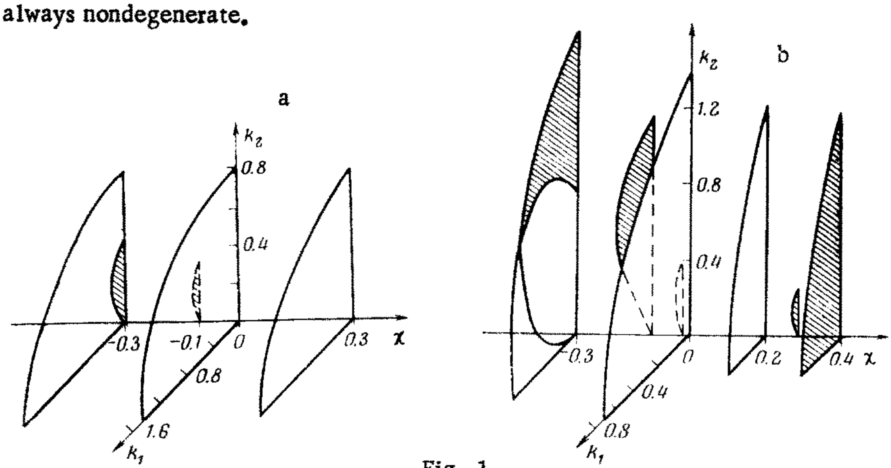

Fig. 1

Another singularity inherent in orthotropic shells of nonnegative Gaussian curvature, appears at the edge with lesser curvature ($\xi_2 = 0$). The edge effect is not degenerate for $\chi \geqslant (E_2 / E_1)^{1/2}$. If $E_2 / E_1 < 1$, then χ are found such that the edge effect will be nondegenerate. Therefore, for $E_1 / E_2 > 1$ there exist values of χ for which the edge effect will be nondegenerate along any of the principal directions.

In this sense, the range of application of the asymptotic method to orthotropic shells is broader than for isotropic shells of corresponding geometry.

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PARAMETRIC VIBRATIONS OF A VISCOELASTIC BAR WITH NONLINEAR HEREDITARY CHARACTERISTIC

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The dynamic stability and resonance modes for parametric vibrations of a viscoelastic bar subjected to a harmonic force are investigated by the method of averaging [1−3]. The connection between the stress and strain is given as the sum of multiple integrals [4−6].

1. Let us consider the problem of transverse vibrations of a rectilinear viscoelastic rod loaded by a force $f(x, t)$ distributed uniformly over the length of the bar, and compressed by a periodic longitudinal force $P(t) = P_0 + P\cos\theta t$. The connection between the stress $\sigma_x(t)$ and the strain $\varepsilon_x(t)$ is expressed by the following nonlinear law

$$\sigma_x(t) = E\left\{\varepsilon_x - \int_0^t R_1(t-\tau)\,\varepsilon_x(\tau)\,d\tau\right\} - \tag{1.1}$$

$$h\int_0^t\int_0^t\int_0^t R_3(t-\tau_1,\ t-\tau_2,\ t-\tau_3)\,\varepsilon_x(\tau_1)\,\varepsilon_x(\tau_2)\,\varepsilon_x(\tau_3)\,d\tau_1 d\tau_2 d\tau_3$$

Here E is the instantaneous elastic modulus, $h > 0$ is a nonlinearity factor, and $R_1(t)$, $R_3(t, t, t)$ are relaxation kernels.

We assume the validity of the law of plane sections and consider a section of the bar to be constant along its length. Then taking account of the appropriate equation for an elastic bar [7, 8], and using the results of [9], we obtain the following integro-differential equation to describe the transverse vibrations of the viscoelastic bar (the prime denotes differentiation with respect to x, and the dot with respect to t):

$$EIU^{\mathrm{IV}} + P(t)\,U'' + mu^{\cdot\cdot} = EI\int_0^t R_1(t-\tau_1)\,U^{\mathrm{IV}}(x,\tau)\,d\tau + \tag{1.2}$$

$$hI_1\int_0^t\int_0^t\int_0^t R_3(t-\tau_1, t-\tau_2, t-\tau_3)[U^{\mathrm{IV}}(x,\tau_1)\,U''(x,\tau_2)\,U''(x,\tau_3) +$$

$$2u'''(x,\tau_1)\,u'''(x,\tau_2)\,u''(x,\tau_3) + 2u'''(x,\tau_1)\,u''(x,\tau_2)\,u'''(x,\tau_3) +$$

$$2u''(x,\tau_1)\,u'''(x,\tau_2)\,u'''(x,\tau_3) + u''(x,\tau_1)\,u^{\mathrm{IV}}(x,\tau_2)\,u''(x,\tau_3) +$$

$$u''(x,\tau_1)\,u''(x,\tau_2)\,u^{\mathrm{IV}}(x,\tau_3)]\,d\tau_1 d\tau_2 d\tau_3 + f(x,t), \qquad I_1 = \int_F z^4 dF$$

Here $u(x, t)$ is the transverse deflection of the bar, m is the mass of the bar per unit length, EI is the bending stiffness, $F = \text{const}$ is the cross-sectional area of the bar and z is the distance between a point of the bar transverse section and the neutral axis.

Considering the bar hinge-supported, we seek the solution of (1.2) which satisfies the boundary conditions of the problem as

$$u(x, t) = T(t)\sin \pi x / l \tag{1.3}$$

where l is the bar length, and $T(t)$ is a still unknown function of the time. Let us now expand the function $f(x, t)$ in a sine series in the argument x in the interval $(0, l)$

$$f(x, t) = \sum_{k=1}^{\infty} F_k(t)\sin\frac{k\pi x}{l}, \qquad F_k(t) = \frac{2}{l}\int_0^l f(x,t)\sin\frac{k\pi x}{l}\,dx \tag{1.4}$$

Substituting (1.3) into (1.2) and taking account of (1.4), we obtain the following equation to determine $T(t)$

$$T^{\cdot\cdot} + p(1 + 2\mu\cos\theta t)\,T = F(t) + \omega^2\int_0^t R_1(t-\tau)\,T(\tau)\,d\tau + \tag{1.5}$$

$$\gamma\int_0^t\int_0^t\int_0^t R_3(t-\tau_1, t-\tau_2,\ t-\tau_3)\,T(\tau_1)\,T(\tau_2)\,T(\tau_3)\,d\tau_1 d\tau_2 d\tau_3$$

$$\omega = (\pi / l)^2 \sqrt{EI / m}, \quad p = \omega \sqrt{1 - P_0 / P_1}, \quad \mu = P_1 / 2 (P_2 - P_0)$$
$$P_2 = (\pi / l)^2 EI, \quad F(t) = F_1(t) / m, \quad \gamma = {}^3/_4 (\pi / l)^8 h I_1 / m$$

Here p is the natural vibrations frequency of a bar loaded by a constant component of the longitudinal force P_0, ω is the natural vibrations frequency of an unloaded bar, and μ is the excitation factor.

We assume that the amplitude of the longitudinal periodic force is a quantity of the order of $\varepsilon : P(t) = P_0 + \varepsilon P_1 \cos \theta t$, and moreover, the bar material possesses the property of low viscosity [6], then

$$\omega^2 R_1(t) = \varepsilon \Gamma(t), \quad \gamma R(t, t, t) = \varepsilon G(t, t, t) \tag{1.6}$$

where $\varepsilon > 0$ is a small parameter. Under the assumptions made, we write (1.5) as

$$T^{\cdot\cdot} + p^2 T = F(t) + \varepsilon \Bigg[2\mu p^2 T \cos \theta t + \int_0^t \Gamma(t-\tau) T(\tau) d\tau + \tag{1.7}$$
$$\int_0^t\int_0^t\int_0^t G(t-\tau_1, t-\tau_2, t-\tau_3) T(\tau_1) T(\tau_2) T(\tau_3) d\tau_1 d\tau_2 d\tau_3 \Bigg]$$

Let us study (1.7) when $F(t) = a \sin \lambda t$ in the case of resonance and non-resonance vibrations.

2. Non-resonance case. Let $p \neq \lambda$. By using the substitution

$$T(t) = c_1 \cos pt + c_2 \sin pt + d \sin \lambda t, \quad d = a / (p^2 - \lambda^2)$$

we reduce (1.7) to a system of equations in the unknowns $c_1 = c_1(t)$, $c_2 = c_2(t)$. The system obtained is solved by the method of averaging [1-3]. For $p \neq \lambda,\ 3\lambda,\ \lambda / 3;$ $\theta \neq 2p,\ p - \lambda,\ p + \lambda,\ \lambda - p$, the averaged system

$$\xi_1^{\cdot} = -\frac{\varepsilon}{2p} \{a_1\xi_1 + a_2\xi_2 + a_3\xi_1(\xi_1^2 + \xi_2^2) + a_4\xi_2(\xi_1^2 + \xi_2^2)\} \tag{2.1}$$
$$\xi_2^{\cdot} = \frac{\varepsilon}{2p} \{a_2\xi_1 - a_1\xi_2 + a_4\xi_1(\xi_1^2 + \xi_2^2) - a_3\xi_2(\xi_1^2 + \xi_2^2)\}$$

corresponds to the system.

Here

$$a_1 = \Gamma_s + d^2 / 2 (A_1 + A_2 + A_3), \quad a_2 = \Gamma_c + d^2 / 2 (B_1 + B_2 + B_3) \tag{2.2}$$
$$a_3 = \frac{1}{4}(C_1 + 2C_2),\ a_4 = \frac{1}{4}(C_3 + 2C_4), \quad \Gamma_s = \int_0^\infty \Gamma(s) \sin psds$$
$$\Gamma_c = \int_0^\infty \Gamma(s) \cos psds, \quad A_1 = \langle \cos \lambda (s_1 - s_2) \sin ps_3 \rangle$$
$$A_2 = \langle \sin ps_1 \cos \lambda (s_2 - s_3) \rangle, \quad A_3 = \langle \cos (s_1 - s_3)\lambda \sin ps_2 \rangle$$
$$B_1 = \langle \cos \lambda (s_1 - s_3) \cos ps_2 \rangle, \quad B_2 = \langle \cos \lambda (s_1 - s_2) \cos ps_3 \rangle$$
$$B_3 = \langle \cos ps_1 \cos \lambda (s_2 - s_3) \rangle, \quad C_1 = \langle \cos p (s_1 + s_2 - s_3) \rangle,$$
$$C_2 = \langle \cos ps_3 \cos p (s_1 - s_2) \rangle, \quad C_3 = \langle \sin p (s_1 + s_2 - s_3) \rangle,$$
$$C_4 = \langle \cos p (s_1 - s_2) \sin ps_3 \rangle$$

Here and henceforth

$$\langle f(s_1, s_2, s_3)\rangle = \int_0^\infty\int_0^\infty\int_0^\infty G(s_1, s_2, s_3)\, f(s_1, s_2, s_3)\, ds_1 ds_2 ds_3$$

The solution of the system (2.1) is

$$\begin{Bmatrix}\xi_1(t)\\ \xi_2(t)\end{Bmatrix} = a_0 \sqrt{a_1/\alpha(t)}\exp\left(-\frac{\varepsilon a_1 t}{2p}\right)\begin{Bmatrix}\sin\\ \cos\end{Bmatrix}\left\{-\frac{\varepsilon a_2 t}{2p} - \frac{a_4}{2a_3}\ln|\alpha(t)| + \varphi_0\right\}$$

$$\alpha(t) = 1 - a_0^2 a_3 \exp\left(-\frac{\varepsilon a_1 t}{p}\right)$$

Here a_0, φ_0 are arbitrary constants determined from the initial conditions. Therefore, on the basis of theorems proved in [1–3], the solution of (1.7) can be approximated for sufficiently small ε and all $t \geqslant 0$ in the case under consideration to any degree of accuracy in ε by the solution of the system (2.1)

$$T(t) \approx \xi_1(t)\cos pt + \xi_2(t)\sin pt + d\sin\lambda t = a_0\sqrt{a_1/\alpha(t)}\, \times \\ \exp(-\varepsilon a_1 t/2p)\sin\left\{\left(p - \frac{\varepsilon a_2}{2p}\right)t - \frac{a_4}{2a_3}\ln|\alpha(t)| + \varphi_0\right\} + d\sin\lambda t \qquad (2.3)$$

Since $a_1 > 0$, then it follows from (2.3) that the vibration amplitude will damp out exponentially in a first approximation when viscosity is present, while the vibration frequency and phase will be shifted depending on the viscoelastic properties of the bar material. As $t \to \infty$, the quantity $T(t)$ tends asymptotically to an expression characterizing the harmonic oscillations

$$T(t) = d\sin\lambda t \qquad (2.4)$$

3. Resonance case. The following kinds of resonances can originate in the system (1.7):

1) $p \neq \lambda,\ 3\lambda,\ \lambda/3,\ \theta = 2p;\ \theta \neq p - \lambda,\ p + \lambda,\ \lambda - p$;
2) $p \neq \lambda,\ 3\lambda,\ \lambda/3;\ \theta = p - \lambda$; 3) $p \neq \lambda,\ 3\lambda,\ \lambda/3;\ \theta = \lambda + p$;
4) $p \neq \lambda,\ 3\lambda,\ \lambda/3;\ \theta = \lambda - p$; 5) $p = 3\lambda;\ \theta = 2\lambda$;
6) $p = 3\lambda;\ \theta = 4\lambda$; 7) $p = 3\lambda;\ \theta \neq 2p,\ p - \lambda,\ p + \lambda,\ \lambda - p$;
8) $p = \lambda/3;\ \theta \neq 2p,\ p - \lambda,\ p + \lambda,\ \lambda - p$; 9) $p = 3\lambda;\ \theta = 6\lambda$;
10) $p = \lambda/3;\ \theta = 2\lambda/3$; 11) $p = \lambda/3;\ \theta = 4\lambda/3$.

Let us investigate the resonance modes listed.

In the case (1) the averaged system

$$\xi_1^{\cdot} = -\frac{\varepsilon}{2p}\{a_1\xi_1 + (a_2 - \mu p^2)\xi_2 + a_3\xi_1(\xi_1^2 + \xi_2^2) + a_4\xi_2(\xi_1^2 + \xi_2^2)\} \qquad (3.1)$$

$$\xi_2^{\cdot} = \frac{\varepsilon}{2p}\{(a_2 + \mu p^2)\xi_1 - a_1\xi_2 + a_4\xi_1(\xi_1^2 + \xi_2^2) - a_3\xi_2(\xi_1^2 + \xi_2^2)\}$$

will correspond to the system (1.7).

It is easy to verify that the point $\xi_1 = \xi_2 = 0$ is the equilibrium position of the system (3.1) which will be asymptotically stable for $\mu^2 p^4 < a_1^2 + a_2^2$ [10]. Therefore, the solution of (1.7) tends asymptotically to the harmonic oscillation (2.4) in the case under consideration.

In cases (2) – (4) we set the averaged system

$$\xi_1^{\cdot} = -\frac{\varepsilon}{2p}\{a_1\xi_1 + a_2\xi_2 + a_3\xi_1(\xi_1^2 + \xi_2^2) + a_4\xi_2(\xi_1^2 + \xi_2^2) + b_j\} \qquad (3.2)$$

$$\xi_2^{\cdot} = \frac{\varepsilon}{2p}\{a_2\xi_1 - a_1\xi_2 + a_4\xi_1(\xi_1^2 + \xi_2^2) - a_3\xi_2(\xi_1^2 + \xi_2^2)\}$$

in correspondence with the system (1.7), where j is the number of the case ($j = 2, 3, 4$), and $b_2 = b_3 = \mu p^2 d$, $b_4 = -\mu p^2 d$. Equating the right sides of the system (3.2) to zero, we obtain the stationary values of ξ_1 and ξ_2

$$\xi_1^\circ = A \cos \varphi, \quad \xi_2^\circ = A \sin \varphi \tag{3.3}$$

Here A and φ are the roots of the system of algebraic equations. The stationary mode (3.3) found is stable if the following inequalities are satisfied [11, 12]

$$a_1 + 2a_3 (\xi_1^{\circ 2} + \xi_2^{\circ 2}) > 0$$

$$a_1^2 + a_2^2 + 4(a_1 a_3 + a_2 a_4)(\xi_1^{\circ 2} + \xi_2^{\circ 2}) + 3(a_3^2 + a_4^2)(\xi_1^{\circ 2} + \xi_2^{\circ 2})^2 > 0$$

In this case (1.7) has the solution $T(t) = \xi_1^\circ \cos pt + \xi_2^\circ \sin pt + d \sin \lambda t$.

Stability of the stationary resonance modes in cases (5) – (8) is investigated analogously, but this question requires further investigation in cases (9) – (11).

4. Now, let us investigate (1.7) near the principal resonance. Let the natural vibrations frequency of the bar p be different from the forced vibrations frequency λ by a quantity proportional to the small parameter ε, and moreover, let the amplitude of the external force a be small of the order of ε, i.e.

$$p^2 = \lambda^2 - \varepsilon q, \quad a = \varepsilon \Delta, \quad q, \ \Delta = \text{const} \tag{4.1}$$

Equation (1.7) becomes by virtue of (4.1)

$$T'' + \lambda^2 T = \varepsilon \Bigg[2\mu p^2 T \cos \theta t + qT + \Delta \sin \lambda t + \int_0^t \Gamma(t-\tau) T(\tau)\, d\tau + \tag{4.2}$$
$$\int_0^t \int_0^t \int_0^t G(t-\tau_1, t-\tau_2, t-\tau_3) T(\tau_1) T(\tau_2) T(\tau_3)\, d\tau_1 d\tau_2 d\tau_3 \Bigg]$$

Introducing new variables by the formula $T(t) = c_1 \cos \lambda t + c_2 \sin \lambda t$, we reduce (4.2) to standard form and by averaging the system obtained for $\theta \neq 2\lambda$, we find

$$\xi_1' = -\frac{\varepsilon}{2\lambda} \{a_1 \xi_1 + (a_2 + q)\xi_2 + a_3 \xi_1 (\xi_1^2 + \xi_2^2) + a_4 \xi_2 (\xi_1^2 + \xi_2^2) + \Delta\} \tag{4.3}$$

$$\xi_2' = \frac{\varepsilon}{2\lambda} \{(a_2 + q)\xi_1 - a_1 \xi_2 + a_4 \xi_1 (\xi_1^2 + \xi_2^2) - a_3 \xi_2 (\xi_1^2 + \xi_2^2)\}$$

where the quantities a_k ($k = 1, 2, 3, 4$) are evaluated by means of (2.2) if λ is substituted in place of p. The stability of the established solutions of the system (4.3) is investigated analogously to the cases (2) – (4).

If $\theta = 2\lambda$, then the behavior of the solution of (4.2) remains open.

5. Let us study the bar vibrations described by (1.7) near the principal parameteric resonance. Setting $p^2 = (\theta / 2)^2 - \varepsilon q_1$, $q_1 = \text{const}$, we reduce (1.7) to the form

$$T'' + \left(\frac{\theta}{2}\right)^2 T = \varepsilon \Bigg[2\mu p^2 T \cos \theta t + q_1 T + \int_0^t \Gamma(t-\tau) T(\tau)\, d\tau + \tag{5.1}$$
$$\int_0^t \int_0^t \int_0^t G(t-\tau_1, t-\tau_2, t-\tau_3) T(\tau_1) T(\tau_2) T(\tau_3)\, d\tau_1 d\tau_2 d\tau_3 \Bigg] + a \sin \lambda t$$

Let $\theta \neq 2\lambda$. Then using the substitution

$$T(t) = c_1 \cos\frac{\theta}{2} t + c_2 \sin\frac{\theta}{2} t + d_1 \sin \lambda t, \quad d_1 = a \bigg/ \left[\left(\frac{\theta}{2}\right)^2 - \lambda^2\right]$$

we reduce (5.1) to standard form and by averaging the system obtained for $\theta \neq 2\lambda$, 6λ, $2\lambda / 3$, we find

$$\xi_1^{\cdot} = -\frac{\varepsilon}{\theta}\{a_1\xi_1 + (a_2 + q_1 - \mu p^2)\xi_2 + a_3\xi_1(\xi_1^2 + \xi_2^2) + a_4\xi_2(\xi_1^2 + \xi_2^2)\} \quad (5.2)$$

$$\xi_2^{\cdot} = \frac{\varepsilon}{\theta}\{(a_2 + q_1 + \mu p^2)\xi_1 - a_1\xi_2 + a_4\xi_1(\xi_1^2 + \xi_2^2) - a_3\xi_2(\xi_1^2 + \xi_2^2)\}$$

where the quantities a_k ($k = 1, 2, 3, 4$) are evaluated be means of (2.2) if $\theta / 2$ is substituted in place of p. It is easy to show that the point $\xi_1 = \xi_2 = 0$ is the equilibrium position of the system (5.2) and this position will be asymptotically stable for $\mu^2 p^4 < a_1^2 + (a_2 + q_1)^2$. In this case the solution of (5.1) therefore tends asymptotically to the harmonic oscillation (2.4).

Stability of the solution of the averaged systems is analyzed analogously in the case $\theta = 2\lambda / 3$.

If $\theta = 6\lambda$, then the behavior of the solution of (5.1) remains open.

6. Now let us turn to an examination of the more general equation (1.5). Setting $F(t) = \varepsilon a \sin \lambda t$ here and taking account of (1.6), we write (5.1) as

$$T^{\cdot\cdot} + p^2(1 - 2\mu\cos\theta t)T = \varepsilon\left[a \sin\lambda t + \int_0^t \Gamma(t-\tau)T(\tau)\,d\tau + \int_0^t\int_0^t\int_0^t G(t-\tau_1, t-\tau_2, t-\tau_3)T(\tau_1)T(\tau_2)T(\tau_3)\,d\tau_1 d\tau_2 d\tau_3\right] \quad (6.1)$$

For $\varepsilon = 0$ Eq. (6.1) degenerates into the known Mathieu equation. We seek its solution in the form

$$T(t) = c_1 y_1(t) + c_2 y_2(t) \quad (6.2)$$

where $y_1(t)$ and $y_2(t)$ are linearly independent particular solutions of the Mathieu equation. It is known [13] that the general solution of the Mathieu equation depends on the magnitude of the characteristic index ν. For imaginary values of ν ($\nu = i\beta$, $0 \leqslant \beta < \theta / 2$) we can set

$$y_1(t) = \sum_{-\infty}^{\infty} H_k \cos(k\theta + \beta)t, \qquad y_2(t) = \sum_{-\infty}^{\infty} H_k \sin(k\theta + \beta)t \quad (6.3)$$

Substituting (6.2) and (6.3) into (6.1) and averaging the system obtained for $k\theta + \beta \neq \lambda$ (k is any integer), we find

$$\xi_1^{\cdot} = -\frac{\varepsilon}{2}\{\alpha_1\xi_1 + \alpha_2\xi_2 + \alpha_3\xi_1(\xi_1^2 + \xi_2^2) + \alpha_4\xi_2(\xi_1^2 + \xi_2^2)\} \quad (6.4)$$

$$\xi_2^{\cdot} = \frac{\varepsilon}{2}\{\alpha_2\xi_1 - \alpha_1\xi_2 + \alpha_4\xi_1(\xi_1^2 + \xi_2^2) - \alpha_3\xi_2(\xi_1^2 + \xi_2^2)\}$$

$$\alpha_1 = \sum_{-\infty}^{\infty} H_k^2 \Gamma_{sk}, \qquad \alpha_2 = \sum_{-\infty}^{\infty} H_k^2 \Gamma_{ck}, \qquad \alpha_3 = \frac{1}{4}\sum_{-\infty}^{\infty} H_k^4 (C_{1k} + 2C_{2k})$$

$$\alpha_4 = \frac{1}{4}\sum_{-\infty}^{\infty} H_k^4 (C_{3k} + 2C_{4k})$$

Here Γ_{sk}, Γ_{ck}, C_{1k}, C_{2k}, C_{3k}, C_{4k} are evaluated by means of (2.2) if $k\theta + \beta$ is substituted in place of p. According to theorems on averaging, the solution of (6.1) for sufficiently small ε and for all $t \geqslant 0$ can be approximated by the expression

$$T(t) \approx \xi_1(t) y_1(t) + \xi_2(t) y_2(t) = a_0 \sqrt{\alpha_1/\delta(t)} \exp(-\varepsilon\alpha_1 t/2) \times \quad (6.5)$$

$$\sum_{-\infty}^{\infty} H_k \sin \{(k\theta + \beta - \varepsilon\alpha_2/2)\, t - \alpha_4/2\alpha_3 \ln |\, \delta(t)\, | + \varphi_0\}$$

where a_0, φ_0 are constants of integration. Since $\alpha_1 > 0$, then it follows from (6.5) that the solution obtained is damped, i.e. is asymptotically stable.

Let us assume that $k\theta + \beta = \lambda$, holds for fixed values of k, then the averaged system

$$\xi_1^{\cdot} = -\frac{\varepsilon}{2}\{\alpha_1\xi_1 + \alpha_2\xi_2 + \alpha_3\xi_1(\xi_1^2 + \xi_2^2) + \alpha_4\xi_2(\xi_1^2 + \xi_2^2) + a\}$$

$$\xi_2^{\cdot} = \frac{\varepsilon}{2}\{\alpha_2\xi_1 - \alpha_1\xi_2 + \alpha_4\xi_1(\xi_1^2 + \xi_2^2) - \alpha_3\xi_2(\xi_1^2 + \xi_2^2)\}$$

will correspond to the system (6.1).

We note that the general form of this system agrees with the system (3.2), hence its investigation will reduce to that elucidated.

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